NOAA Technical Memorandum ERL WPL-225

GUIDE TO MICROWAVE WEIGHTING FUNCTION CALCULATIONS

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Boulder, Colorado
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ABSTRACT. This document describes the theory and numerical algorithms that we use to compute weighting functions for an upward-looking microwave radiometer at a given channel frequency and antenna orientation. We use these weighting functions to assess the potential response of hypothetical radiometer systems to changes in atmospheric temperature, pressure, water vapor density, dry air density, and cloud liquid density, given an atmosphere defined by input vertical profiles of pressure, temperature, humidity, and cloud density. Calculations for off-zenith antenna orientations assume a spherically stratified atmosphere. The radiative transfer model that we use is valid for channel frequencies below 1 THz in clear conditions and for frequencies below 100 GHz when clouds are present.

1. INTRODUCTION

The NOAA Wave Propagation Laboratory (WPL) Thermodynamic Profiling Program designs, develops, and operates upward-looking microwave radiometer systems to monitor atmospheric temperature, pressure, humidity, and cloud liquid water content. A weighting function for a specific atmospheric parameter gives the theoretical change in radiometer brightness temperature (T_b) that results from a unit change in that parameter at a given height. The weighting functions are specific to the radiometer channel frequency and antenna orientation. Calculating and examining weighting functions for proposed radiometric channels allows one to estimate the system response to atmospheric parameters of interest before investing resources in its construction. This document describes the theory and numerical algorithms that WPL uses to compute weighting functions for upward-looking microwave radiometers. Our present software computes weighting functions for temperature, pressure, water vapor density, dry air density, and cloud liquid water density for any combination of microwave channel frequency and antenna elevation angle.

Figure 1 illustrates graphically the information that weighting functions provide. Each frame shows a set of weighting functions for a different atmospheric parameter. Each curve represents the response of a single channel of a ground-based, zenith-pointing microwave radiometer to a unit change in the respective atmospheric parameter over a 1-km layer at various heights above the radiometer. For example, Fig. 1a implies that a 1-K change in the surface layer temperature produces about a 3-K change in T_b

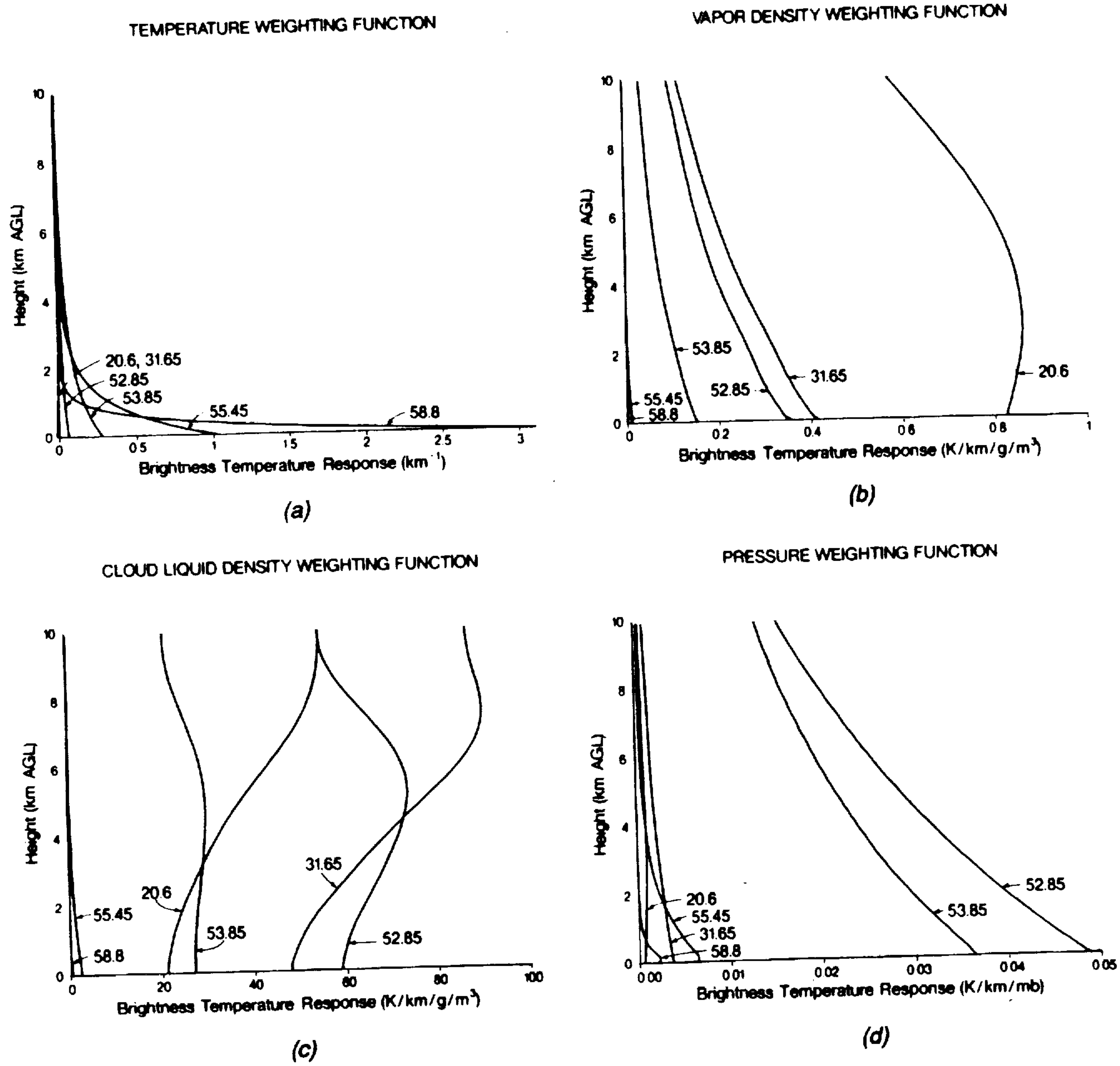

Fig. 1. Weighting functions for (a) temperature, (b) water vapor density, (c) cloud liquid density, and (d) pressure for a ground-based, zenith-pointing microwave radiometer with the channel frequencies shown (GHz). The weighting functions were computed from mean winter profiles of pressure, temperature, and water vapor density at Denver, Colorado. Cloud liquid weighting functions represent radiometer response to a liquid cloud forming in a clear atmosphere.

at 58.8 GHz, a 1-K change in T_b at 55.45 GHz, and no change at 20.6 and 31.65 GHz. A 1-K change in temperature at 2 km AGL would not affect the 58.80-GHz channel, but the T_b at 53.85 and 55.45 GHz would change by about 0.2 K. All of the weighting functions shown in Fig. 1 were calculated from mean winter profiles of pressure, temperature, and water vapor density computed from ten years of twice-daily radiosonde observations taken at Denver, Colorado during December, January, and February.

By definition, the weighting function for a given atmospheric parameter requires all other atmospheric parameters to be held constant. The humidity weighting functions shown in Fig. 1b were computed in terms of water vapor density rather than relative humidity, because relative humidity depends on temperature as well as water vapor. The cloud liquid density weighting functions shown in Fig. 1c were obtained by adding 1 gm^{-3} of liquid water at a given height to a cloud liquid profile representing a clear atmosphere (10^{-8} gm^{-3} of liquid water everywhere to facilitate calculations). Different cloud weighting functions would have resulted if cloud(s) had been modeled in the profiles used in the calculations. The actual units for the temperature weighting functions shown in Fig. 1a are K km^{-1} K^{-1}, which reduces to the units shown.

Our weighting function calculations use the theory of radiative transfer from a non-scattering atmosphere at microwave frequencies to compute the T_b that would be observed, given an atmosphere whose composition is completely defined by user-supplied profiles of pressure, temperature, humidity, and cloud density. Calculations for off-zenith antenna orientations assume a spherically stratified atmosphere. The model of Liebe and Layton (1987) used to compute water vapor and oxygen absorption is valid for frequencies up to 1 THz. The Rayleigh approximation that underlies the cloud absorption algorithm (Westwater, 1974) restricts its validity to nonprecipitating clouds with particle radii less than about 100 μm; scattering is neglected. Therefore, the cloud weighting functions are valid only for frequencies less than 100 GHz.

Section 2 gives the theory and assumptions that underlie the weighting function calculations, and Section 3 describes the numerical implementation of that theory. Our weighting function software is a user interface to the UNIX Fortran radiative transfer library that Reynolds and Schroeder (1992) developed from the radiative transfer algorithms and software described by Schroeder and Westwater (1991).

2. THEORETICAL BASIS

This section describes the theory and assumptions behind the weighting function calculations. Some of the equations presented here will be referenced by number in Section 3, which describes the numerical implementation of this theory.

2.1. Defining the Microwave Brightness Temperature (T_b)

Assuming local thermodynamic equilibrium, the radiative intensity (I), or radiance, at a given frequency for a blackbody radiator is given by the Planck function (Goody and Yung, 1989)

$$B(T) = \left(\frac{2h\nu^3}{c^2}\right)\left(\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right), \tag{1}$$

where h = Planck constant
ν = frequency
c = speed of light
k = Boltzmann constant
T = temperature.

I is often expressed as an equivalent blackbody temperature, or brightness temperature, T_b, such that $I = B(T_b)$. A radiometer antenna looking upward toward cold space receives microwave radiation from two sources: the cosmic background and the atmosphere itself. If discrete sources, such as the sun or moon, are outside the radiometer field of view, the cosmic radiation can be treated as a uniform background. At microwave frequencies, atmospheric scattering can be ignored for non-precipitating conditions, so that the propagating radiation is modified by absorption alone. With these two assumptions, we can express the T_b observed at a given channel frequency and elevation angle as the sum of two terms:

$$B(T_b) = B(T_{bg})\, e^{-\tau(0,\infty)} + \int_0^{\infty} B(T(s))\, \alpha(s)\, e^{-\tau(0,s)}\, ds, \tag{2}$$

where T_b = brightness temperature
T_{bg} = cosmic background temperature
s = arc length along the refracted ray path
$T(s)$ = temperature of layer between s and $s + ds$
$\alpha(s)$ = absorption of layer between s and $s + ds$

and

$$\tau(a,b) = \int_{s(a)}^{s(b)} \alpha(s)\, ds. \tag{3}$$

The factor $e^{-\tau(a,b)}$ in each term represents the exponential decay of the source radiation as it is attenuated by the layer of atmosphere between points a and b, where 0 represents the antenna position. The first term in (2) represents the cosmic background radiation, attenuated by the entire atmosphere as a single layer. The second term in (2) represents the sum of the radiation contributions from an infinite number of atmospheric layers. Each layer's contribution is attenuated by the layers between it and the antenna.

When we substitute the Planck function defined in (1) back into the three places that it appears in (2), the ratio $2h\nu^3c^{-2}$ falls out of the equation. Therefore, we define a modified Planck function, $\tilde{B}(T)$, to simplify the software (Section 3) as

$$\tilde{B}(T) = \frac{1}{e^{\frac{h\nu}{kT}} - 1}, \tag{4}$$

where h, ν, k, and T were defined in (1).

2.2. Defining the Weighting Function

We define the weighting function for atmospheric parameter x at height z for a given channel frequency and elevation angle as

$$W_x(z) = \lim_{\substack{\delta x \to 0 \\ \delta z \to 0}} \frac{\delta T_b}{\delta x \delta z} = \lim_{\substack{\delta x \to 0 \\ \delta z \to 0}} \frac{\delta T_b}{\delta \tilde{B}(T_b)} \frac{\delta \tilde{B}(T_b)}{\delta x \delta z}, \tag{5}$$

where $W_x(z)$ = weighting function for parameter x at height z.
δx = perturbation in atmospheric parameter x
δz = layer thickness at height z
δT_b = resulting change in brightness temperature
$\delta \tilde{B}(T_b)$ = resulting change in the modified Planck radiance [see (4)].

2.2.1. Computing Factor 1: $\frac{\delta T_b}{\delta \tilde{B}(T_b)}$

We obtain the first factor in (5) directly by solving (4) for T_b and differentiating as follows:

$$\frac{\delta T_b}{\delta \tilde{B}(T_b)} = \frac{-h\nu\left\{k\left(\frac{1}{[\tilde{B}(T_b)]^{-1}+1}\right)\left(-[\tilde{B}(T_b)]^{-2}\right)\right\}}{\left(k \ln\left\{[\tilde{B}(T_b)]^{-1}+1\right\}\right)^2}$$

$$\frac{\delta T_b}{\delta \bar{B}(T_b)} = \frac{h\nu k}{[\bar{B}(T_b)]^2 \left\{[\bar{B}(T_b)]^{-1} + 1\right\} k^2 \left(\ln\left\{[\bar{B}(T_b)]^{-1} + 1\right\}\right)^2}$$

$$\frac{\delta T_b}{\delta \bar{B}(T_b)} = \frac{h\nu}{k\left\{\bar{B}(T_b) + [\bar{B}(T_b)]^2\right\}\left(\ln\left\{[\bar{B}(T_b)]^{-1} + 1\right\}\right)^2} \ . \tag{6}$$

2.2.2. *Computing Factor 2:* $\frac{\delta \bar{B}(T_b)}{\delta x \, \delta z}$

The second factor in (5) represents the change in $\bar{B}(T_b)$ that results when an atmospheric parameter x is perturbed by an amount δx over a thin layer of thickness δz with refracted path length δs (Fig. 2). The change in $\bar{B}(T_b)$ is caused primarily by the resulting change in absorption, which can be expressed as

$$\delta\alpha = \frac{\partial\alpha}{\partial x}\,\delta x, \tag{7}$$

where α and δx were defined in (2) and (5), respectively. When temperature is the parameter perturbed, $\bar{B}(T_b)$ is also affected by the change in $\bar{B}(T)$ [see (25) and (26)].

We use the radiative transfer equations (2) and (3) to compute factor 2. However, since (5) defines the weighting function in terms of height (z), we choose to rewrite (2) and (3) in terms of z rather than s to simplify the derivations in this section. Assuming a spherically-stratified atmosphere, each coordinate z lies on a spherical shell along which the atmospheric parameters are constant. Then coordinate s represents the arc length along the refracted ray path from the radiometer antenna to the shell on which z lies. For example, consider layer B in Fig. 2 as defined by the two concentric shells on which z_1 and z_2 lie. For both antenna orientations shown, the s coordinate corresponding to z_1 (call it s_1) is the distance along the respective ray path from the antenna to the shell on which z_1 lies. For the zenith antenna, $s_1 = z_1$. For any off-zenith antenna, $s_1 > z_1$, and s_1 changes with the antenna elevation angle and the atmospheric refractive index profile. For this spherically-stratified atmosphere, $B(T(s))$ and $\alpha(s)$ in (2) and (3) have the same values as $B(T(z))$ and $\alpha(z)$, respectively. To change the variable of integration in (2) and (3) from z to s, we introduce the change of variable

$$\bar{\alpha}(z) = \alpha(s)\frac{ds}{dz}\ , \tag{8}$$

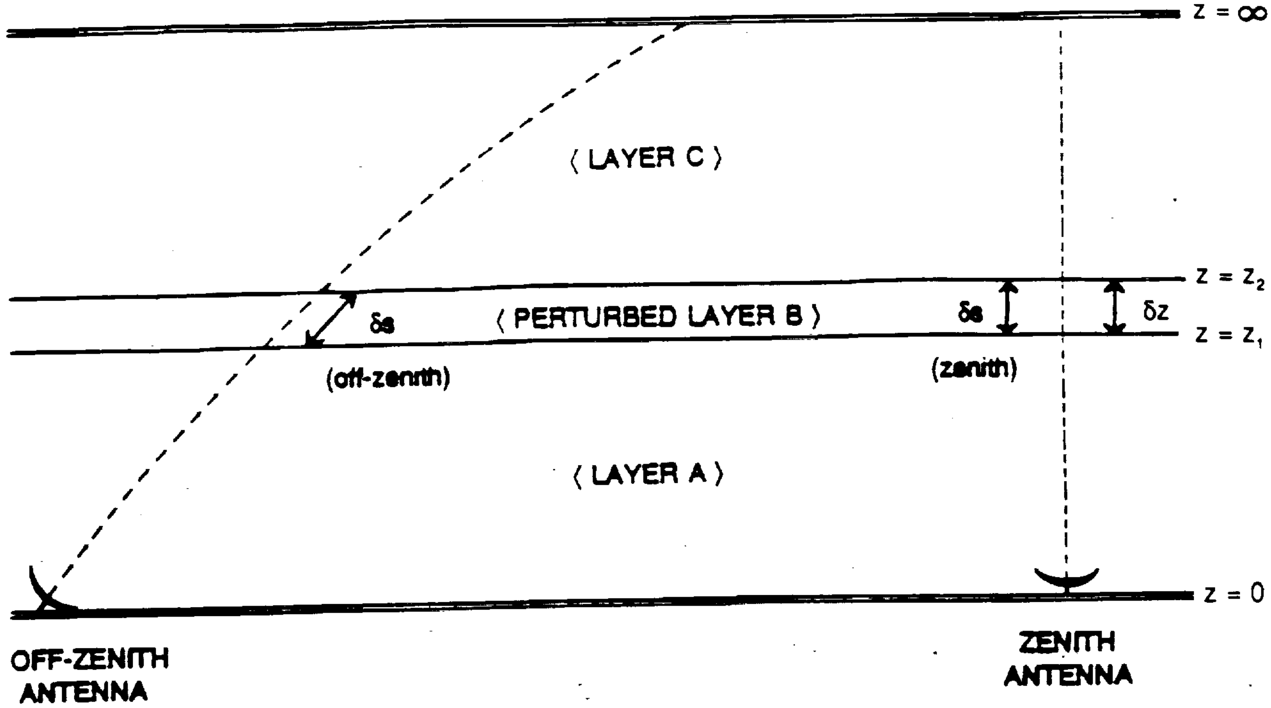

Fig. 2. Schematic of the atmosphere, divided into three layers. Between heights z_1 and z_2 (layer B), we perturb atmospheric parameter x by an amount δx. The dashed lines represent the ray paths for antennas pointed in the zenith and off-zenith directions.

where $\frac{ds}{dz}$ is the derivative of arc length with respect to height. The assumptions discussed above and the definitions in (4) and (8) allow us to express the integrals in (2) and (3) in terms of height (z) instead of arc length (s) as follows:

$$\tilde{B}(T_b) = \tilde{B}(T_{bg})\, e^{-\tau(0,\infty)} + \int_0^\infty \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\tau(0,z)}\, dz, \tag{2'}$$

where

$$\tau(a,b) = \int_{z(a)}^{z(b)} \tilde{\alpha}(z)\, dz. \tag{3'}$$

Equipped with these definitions, we first determine the contribution of the cosmic background term in (2′) to $\delta\tilde{B}(T_b)$ in (5). We can express the perturbed form of that term as

$$\tilde{B}(T_{bg})\, e^{-\left(\int_0^\infty \tilde{\alpha}\, dz + \int_{z_1}^{z_2} \delta\tilde{\alpha}\, dz\right)},$$

which is equivalent to

$$\bar{B}(T_{bg})\, e^{-\int_0^{\infty} \bar{\alpha}\, dz}\, e^{-\int_{z_1}^{z_2} \delta\bar{\alpha}\, dz} . \tag{9}$$

Since $\delta\bar{\alpha}\,(z_2 - z_1)$ is small, we can use the approximation $e^{-x} \approx 1-x$ to express (9) as

$$\bar{B}(T_{bg})\, e^{-\int_0^{\infty} \bar{\alpha}\, dz} \left(1 - \int_{z_1}^{z_2} \delta\bar{\alpha}\, dz \right) . \tag{10}$$

Using the definition of τ from (3′), we can express the background contribution to $\delta\bar{B}(T_b)$ as the difference between (10) and the unperturbed background term in (2′):

$$\bar{B}(T_{bg})\, e^{-\tau(0,\infty)} \left(1 - \int_{z_1}^{z_2} \delta\bar{\alpha}\, dz \right) - \bar{B}(T_{bg})\, e^{-\tau(0,\infty)} ,$$

which is equivalent to

$$- \bar{B}(T_{bg})\, e^{-\tau(0,\infty)} \int_{z_1}^{z_2} \delta\bar{\alpha}\, dz . \tag{11}$$

The contribution of the atmospheric term in (2′) to $\delta\bar{B}(T_b)$ is far more complicated. To find this quantity, we express the integral in (2′) as the sum of three integrals, which correspond to the three atmospheric layers shown in Fig. 2:

$$\begin{aligned} \int_0^{\infty} \bar{B}(T(z))\, \bar{\alpha}(z)\, e^{-\int_0^z \bar{\alpha} dz'} dz = & \int_0^{z_1} \bar{B}(T(z))\, \bar{\alpha}(z)\, e^{-\int_0^z \bar{\alpha} dz'} dz \\ & + \int_{z_1}^{z_2} \bar{B}(T(z))\, \bar{\alpha}(z)\, e^{-\int_0^z \bar{\alpha} dz'} dz \\ & + \int_{z_2}^{\infty} \bar{B}(T(z))\, \bar{\alpha}(z)\, e^{-\int_0^z \bar{\alpha} dz'} dz . \end{aligned} \tag{12}$$

The integral corresponding to layer A is not affected by the perturbation of layer B, so its contribution to $\delta\bar{B}(T_b)$ is zero.

We can express the integral that corresponds to layer B equivalently as

$$\int_{z_1}^{z_2} \bar{B}(T(z))\,\bar{\alpha}(z)\,e^{-\int_0^z \bar{\alpha}\,dz'}\,dz = \int_{z_1}^{z_2} \bar{B}(T(z))\,\bar{\alpha}(z)\,e^{-\left(\int_0^{z_1} \bar{\alpha}\,dz' + \int_{z_1}^{z} \bar{\alpha}\,dz'\right)}\,dz.$$

Pulling the portion of the exponential term that belongs to layer A outside of the integral puts the unperturbed contribution of layer B into the following form:

$$e^{-\int_0^{z_1} \bar{\alpha}\,dz} \int_{z_1}^{z_2} \bar{B}(T(z))\,\bar{\alpha}(z)\,e^{-\int_{z_1}^{z} \bar{\alpha}\,dz'}\,dz. \tag{13}$$

After perturbing x by δx within layer B, (13) becomes

$$e^{-\int_0^{z_1} \bar{\alpha}\,dz} \int_{z_1}^{z_2} [\bar{B} + \delta\bar{B}(T(z))]\,(\bar{\alpha} + \delta\bar{\alpha}(z))\,e^{-\int_{z_1}^{z} (\bar{\alpha}+\delta\bar{\alpha})\,dz'}\,dz,$$

which is equivalent to

$$e^{-\int_0^{z_1} \bar{\alpha}\,dz} \int_{z_1}^{z_2} [\bar{B} + \delta\bar{B}(T(z))]\,(\bar{\alpha} + \delta\bar{\alpha}(z))\,e^{-\int_{z_1}^{z} \bar{\alpha}\,dz'}\,e^{-\int_{z_1}^{z} \delta\bar{\alpha}\,dz'}\,dz. \tag{14}$$

Since $\int_{z_1}^{z} \delta\bar{\alpha}\,dz'$ is small, we use the approximation $e^{-x} \approx 1-x$ to express (14) as

$$e^{-\int_0^{z_1} \bar{\alpha}\,dz} \int_{z_1}^{z_2} [\bar{B} + \delta\bar{B}(T(z))]\,(\bar{\alpha} + \delta\bar{\alpha}(z))\,e^{-\int_{z_1}^{z} \bar{\alpha}\,dz'}\left(1 - \int_{z_1}^{z} \delta\bar{\alpha}\,dz'\right)dz. \tag{15}$$

Expanding (15) gives

$$e^{-\int_0^{z_1} \bar{\alpha}\,dz}\left[\int_{z_1}^{z_2} [\bar{B} + \delta\bar{B}(T(z))]\,(\bar{\alpha} + \delta\bar{\alpha}(z))\,e^{-\int_{z_1}^{z} \bar{\alpha}\,dz'}\,dz - \int_{z_1}^{z_2} [\bar{B} + \delta\bar{B}(T(z))]\,(\bar{\alpha} + \delta\bar{\alpha}(z))\,e^{-\int_{z_1}^{z} \bar{\alpha}\,dz'}\int_{z_1}^{z} \delta\bar{\alpha}\,dz'\,dz\right],$$

which expands further into

$$e^{-\int_0^{z_1}\bar{\alpha}\,dz}\left(\int_{z_1}^{z_2}[\bar{B}+\delta\bar{B}(T(z))](\bar{\alpha}(z))\,e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\,dz+\int_{z_1}^{z_2}[\bar{B}+\delta\bar{B}(T(z))]\,\delta\bar{\alpha}(z)\,e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\,dz\right.$$

$$\left.-\int_{z_1}^{z_2}[\bar{B}+\delta\bar{B}(T(z))]\bar{\alpha}(z)\,e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\int_{z_1}^{z}\delta\bar{\alpha}\,dz'\,dz-\int_{z_1}^{z_2}[\bar{B}+\delta\bar{B}(T(z))]\delta\bar{\alpha}(z)\,e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\int_{z_1}^{z}\delta\bar{\alpha}\,dz'\,dz\right). \quad (16)$$

Since $\delta\bar{\alpha}\,(z_2-z_1)$ is small, the two terms in (16) that contain $\delta\bar{\alpha}\,dz'$ will vanish. The part of the equation that remains represents the radiance contribution from a perturbed layer B. The difference between that expression and the unperturbed expression for layer B given in (13) represents the contribution of layer B to $\delta\bar{B}(T_b)$:

$$e^{-\int_0^{z_1}\bar{\alpha}\,dz}\int_{z_1}^{z_2}e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\left\{[\bar{B}+\delta\bar{B}(T(z))]\,\bar{\alpha}(z))+[\bar{B}+\delta\bar{B}(T(z))]\,\delta\bar{\alpha}(z)-\bar{B}(T(z))\,\bar{\alpha}(z)\right\}dz. \quad (17)$$

Since the term $\delta\bar{B}(T(z))\,\delta\bar{\alpha}(z)$ goes to zero, (17) will reduce to

$$e^{-\int_0^{z_1}\bar{\alpha}\,dz}\int_{z_1}^{z_2}e^{-\int_{z_1}^{z}\bar{\alpha}\,dz'}\left[\delta\bar{B}(T(z))\,\bar{\alpha}(z))+\bar{B}(T(z))\,\delta\bar{\alpha}(z)\right]dz.$$

Using the Mean Value Theorem and the definition of τ from (3′), we can express layer B's contribution to $\delta\bar{B}(T_b)$ as

$$e^{-[\tau(0,z_1)+\bar{\tau}(z_1,z_2)]}\left(\int_{z_1}^{z_2}\delta\bar{B}(T(z))\,\bar{\alpha}(z)\,dz+\bar{B}(\bar{T}(z_1,z_2))\int_{z_1}^{z_2}\delta\bar{\alpha}(z)\,dz\right). \quad (18)$$

Finally, to find layer C's contribution to $\delta\bar{B}(T_b)$, we express the third term in (12) equivalently as

$$\int_{z_2}^{\infty}\bar{B}(T(z))\,\bar{\alpha}(z)\,e^{-\left(\int_0^{z_2}\bar{\alpha}\,dz'+\int_{z_2}^{z}\bar{\alpha}\,dz'\right)}dz,$$

so that the unperturbed contribution of layer C to the atmospheric radiation becomes

$$e^{-\int_0^{z_2} \tilde{\alpha}\, dz} \int_{z_2}^{\infty} \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\int_{z_2}^{z} \tilde{\alpha}\, dz'} dz. \tag{19}$$

After we perturb layer B by δx, (19) looks like

$$e^{-\left(\int_0^{z_2} \tilde{\alpha}\, dz + \int_{z_1}^{z_2} \delta\tilde{\alpha}\, dz\right)} \int_{z_2}^{\infty} \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\int_{z_2}^{z} \tilde{\alpha}\, dz'} dz. \tag{20}$$

Note that the integral from z_2 to infinity in (20) is unaffected by the perturbation of layer B. Now the difference between the perturbed (20) and unperturbed (19) expressions for layer C becomes

$$e^{-\int_0^{z_2} \tilde{\alpha}\, dz} \int_{z_2}^{\infty} \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\int_{z_2}^{z} \tilde{\alpha}\, dz'} dz \left(e^{-\int_{z_1}^{z_2} \delta\tilde{\alpha}\, dz} - 1 \right).$$

Since the quantity $\delta\tilde{\alpha}(z_2 - z_1)$ is small, we again use the approximation $e^{-x} \approx 1-x$ to express the contribution of layer C to $\delta\tilde{B}(T_b)$ as

$$- e^{-\tau(0,z_2)} \int_{z_1}^{z_2} \delta\tilde{\alpha}\, dz \int_{z_2}^{\infty} \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\tau(z_2,z)} dz, \tag{21}$$

where τ was defined in (3′).

The total atmospheric contribution to $\delta\tilde{B}(T_b)$ from layers A, B, and C is the sum of (18) and (21). Combining that result with the background contribution described by (11) gives

$$\delta\tilde{B}(T_b) = \int_{z_1}^{z_2} \delta\tilde{\alpha}\, dz \left\{ -\tilde{B}(T_{bg})\, e^{-\tau(0,\infty)} + \tilde{B}(\bar{T}(z_1,z_2))\; e^{-[\tau(0,z_1) + \bar{\tau}(z_1,z_2)]} - e^{-\tau(0,z_2)} \int_{z_2}^{\infty} \tilde{B}(T(z))\, \tilde{\alpha}(z)\, e^{-\tau(z_2,z)}\, dz \right\} + e^{-[\tau(0,z_1) + \bar{\tau}(z_1,z_2)]} \int_{z_1}^{z_2} \delta\tilde{B}(T(z))\, \tilde{\alpha}(z)\, dz. \tag{22}$$

To obtain the factor $\frac{\delta\tilde{B}(T_b)}{\delta x \delta z}$ that appears in (5), we divide both sides of (22) by $\delta x \delta z$. When we do, the factor outside the braces on the righthand side of (22) becomes

$$\frac{\int_{z_1}^{z_2} \delta\bar{\alpha}\, dz}{\delta x \delta z} = \frac{\overline{\delta\bar{\alpha}}\,(z_2 - z_1)}{\delta x \delta z} = \frac{\overline{\delta\bar{\alpha}}}{\delta x}$$

by the Mean Value Theorem. Similarly, the term outside the braces in (22) becomes

$$e^{-[\tau(0,z_1) + \bar{\tau}(z_1,z_2)]}\ \frac{\overline{\delta\bar{B}(T)}}{\delta x}\ \frac{\bar{\bar{\alpha}}}{\delta z}\ (z_2 - z_1) = \bar{\bar{\alpha}}\ e^{-[\tau(0,z_1) + \bar{\tau}(z_1,z_2)]}\ \frac{\overline{\delta\bar{B}(T)}}{\delta x}\ .$$

In the limit, as δz approaches zero, $z_1 = z_2 = z$, and the bars indicating layer means disappear. Using the relationship shown in (7),

$$\frac{\delta\bar{\alpha}}{\delta x} = \frac{\left(\frac{\partial\bar{\alpha}}{\partial x} \cdot \delta x\right)}{\delta x} = \frac{\partial\bar{\alpha}}{\partial x}\ . \tag{23}$$

Using the same logic,

$$\frac{\delta\bar{B}(T)}{\delta x} = \frac{\partial\bar{B}(T)}{\partial x}\ .$$

Incorporating these ideas into (22) gives the following form for factor 2:

$$\lim_{\substack{\delta x \to 0 \\ \delta z \to 0}} \frac{\delta\bar{B}(T_b)}{\delta x \delta z} = \frac{\partial\bar{\alpha}}{\partial x}\left\{-\bar{B}(T_{bg})\, e^{-\tau(0,\infty)} + \bar{B}(T(z))\ e^{-\tau(0,z)} - \int_z^{\infty} \bar{B}(T(z))\, \bar{\alpha}(z)\, e^{-\tau(0,z')} dz'\right\}$$
$$+\ \bar{\alpha}(z)\, e^{-\tau(0,z)}\ \frac{\partial\bar{B}(T(z))}{\partial x}\ . \tag{24}$$

After reversing the change of variables implemented at the beginning of this section, the product of factor 1 [eq.(6)] and factor 2 [eq.(24)] gives the form of (5) that we use to compute the weighting function for atmospheric parameter x at height z for a given channel frequency and antenna orientation of an upward-looking microwave radiometer:

$$W_x(z) = \left(\frac{h\nu}{k\{\bar{B}(T_b) + [\bar{B}(T_b)]^2\}\,(\ln\{[\bar{B}(T_b)]^{-1} + 1\})^2}\right) *$$
$$\frac{ds}{dz}\left(\frac{\partial\alpha}{\partial x}\left\{-\bar{B}(T_{bg})\, e^{-\tau(0,\infty)} + \bar{B}(T(s))\ e^{-\tau(0,s)} - \int_s^{\infty} \bar{B}(T(s))\, \alpha(s)\, e^{-\tau(0,s')} ds'\right\}\right.$$
$$\left. +\ \alpha(s)\, e^{-\tau(0,s)}\ \frac{\partial\bar{B}(T(s))}{\partial x}\right), \tag{25}$$

where $W_x(z)$ = weighting function for parameter x at height z.
x = atmospheric parameter (e.g., temperature)
z = height above radiometer antenna
h = Planck constant
ν = frequency
k = Boltzmann constant
$\tilde{B}(\,)$ = modified Planck function [see (4)]
T_b = brightness temperature
T_{bg} = cosmic background temperature
s = arc length along the refracted ray path
$\alpha(s)$ = absorption of layer between s and $s + ds$
$T(s)$ = temperature of layer between s and $s + ds$
$\tau(a,b)$ = α integrated between coordinates a and b [see (3)].

The last term in (25), which contains the partial derivative of (4) with respect to atmospheric parameter x, is zero for all choices of x except for temperature. For temperature, that term becomes

$$\alpha(s)\, e^{-\tau(0,s)} \frac{h\nu\, e^{\frac{h\nu}{kT}}}{kT^2 \left(e^{\frac{h\nu}{kT}} - 1\right)^2} \; . \qquad (26)$$

2.3. Assumptions and Limitations

The radiative transfer software and algorithms referenced in this document were designed to calculate thermal emission and absorption in the troposphere. We assume that discrete radiation sources, such as the sun or moon, are outside the radiometer field of view.

The clear-sky atmospheric absorption model (Liebe and Layton, 1987) is valid for frequencies up to 1 THz. However, we only model absorption from water vapor and oxygen, neglecting absorption from minor constituents, such as ozone. Depending on the concentration of a neglected constituent and on the frequency in question, monochromatic calculations may be incorrect.

The cloud absorption model referenced here (Westwater, 1974) assumes the Rayleigh approximation, under which scattering is negligible relative to absorption, and absorption is independent of cloud particle size distribution. These assumptions restrict the model to nonprecipitating clouds with particle radii less than about 100 μm for frequencies less than 100 GHz. Therefore, the algorithms are not adequate for modeling rain, large droplets, or large ice particles in clouds.

3. NUMERICAL IMPLEMENTATION

This section describes the numerical implementation of the weighting function theory presented in Section 2, where the equation numbers referenced here were defined. We italicize names of variables, arrays, and subroutines for emphasis.

Figure 3 shows the logic that we use to compute weighting functions for a selected set of *nx* atmospheric parameters (array *x*) for an upward-looking microwave radiometer with a given channel frequency (*frq*) and antenna orientation (*angle*). Our present software computes weighting functions for any of the following atmospheric parameters: temperature, pressure, water vapor density, dry air density, and cloud liquid density. Calculations are performed by a UNIX Fortran radiative transfer library (*RTE*) that Reynolds and Schroeder (1992) developed from the radiative transfer algorithms and software described by Schroeder and Westwater (1991). The two subroutine calls shown inside the boxes labeled (1) and (2) in Fig. 3 are user interfaces to that library. Interface *Radiance_RTE* models the transfer of radiation at the input frequency and elevation angle through the atmosphere defined by the input vertical profiles of height, pressure, temperature, vapor pressure, cloud liquid density, and cloud ice density. We assume a spherically stratified atmosphere for off-zenith angles. *Weight_RTE* computes weighting functions for each selected atmospheric parameter *x(i)* from the *Radiance_RTE* outputs, which are independent of the parameter selection.

3.1 The Input Profiles

The atmospheric conditions along the ray path are defined by input vertical profiles of height (array *z*, in km MSL), pressure (array *p*, in mb), temperature (array *tk*, in K), vapor pressure (array *e*, in mb), cloud liquid density (array *denliq*, in gm^{-3}), and cloud ice density (array *denice*, in gm^{-3}). Typically, these profiles come from radiosonde measurements, model atmospheres, climatological means, or the user's imagination. The *RTE* library interfaces do not modify the input profiles in any way. If needed, *RTE* interface *Modify_RTE* can be called to extrapolate the profiles to 50 or 0.1 mb, truncate them to begin at a specified level, or insert levels by interpolating between levels that are separated by more than a specified pressure difference. *Weight_RTE* automatically replaces values of 0.0 in array *denliq* with the value 10^{-8} to facilitate cloud liquid density weighting function calculations.

The shape of the weighting function curves will reflect the shape of the input profiles. For example, the weighting functions shown in Fig. 1a, 1b, and 1d are smooth curves because they were calculated from profiles obtained by averaging ten years of winter radiosonde soundings. Single soundings tend to produce irregular curves. The cloud liquid density weighting functions shown in Fig. 1c resulted from introducing a liquid cloud into a clear sky (10^{-8} gm^{-3} liquid and 0 gm^{-3} ice at all profile levels). Figure 1c would look entirely different if the input profiles had contained appreciable cloud water densities.

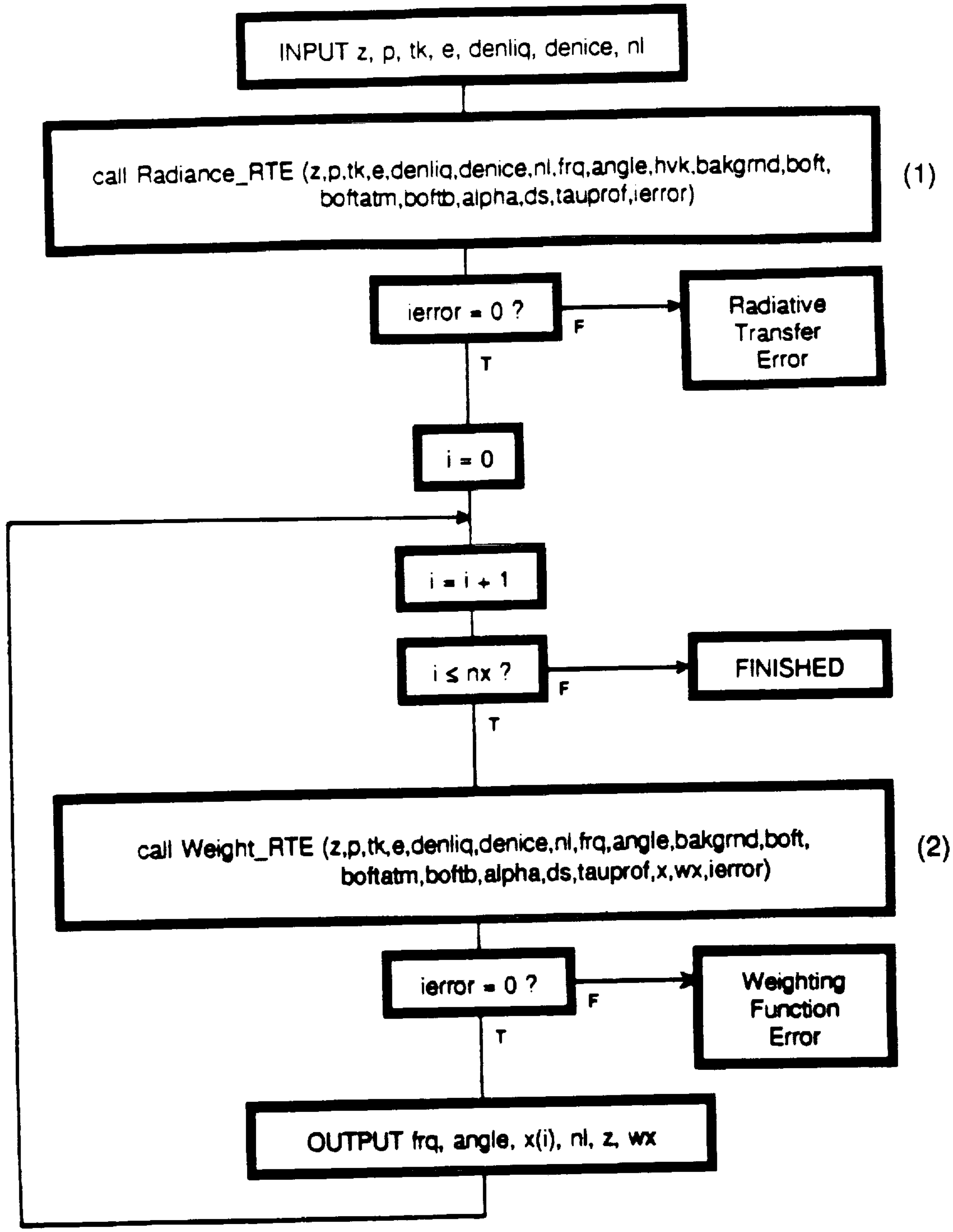

Fig. 3. Flow chart showing logic for computing weighting functions for nx parameters, given a single combination of channel frequency and elevation angle.

3.2 The Radiative Transfer Calculations

Radiance_RTE performs radiative transfer calculations for the ray path defined by *angle* at channel *frq* through the atmosphere defined by the input profiles. It uses the algorithms described by Schroeder and Westwater (1991). The following *Radiance_RTE* outputs are used as inputs to *Weight_RTE*:

bakgrnd	=	cosmic background term from (2)
boft	=	$\tilde{B}(T)$ profile computed from input *tk* profile and (4)
boftatm	=	atmospheric term from (2), where boftatm(i) is the atmospheric term integral from the antenna to profile level i
boftb	=	$\tilde{B}(T_b)$ from (2)
alpha	=	$\tilde{\alpha}(s)$ profile computed from input profiles
ds	=	profile of refracted path length between profile levels (*ds(1)* $\equiv$ 0)
tauprof	=	profile of τ(0,s(i)) computed from (3).

3.3 The Weighting Function Calculations

Weight_RTE computes the weighting function for a specified atmospheric parameter, channel frequency, and elevation angle from the input profiles and the quantities output by *Radiance_RTE* listed above. The input value for variable *x* selects the parameter as follows:

1) temperature
2) pressure
3) water vapor density
4) dry air density
5) cloud liquid density.

Weight_RTE automatically replaces values of 0.0 in array *denliq* with the value 10^{-8} to facilitate cloud liquid density weighting function calculations.

Weight_RTE computes the weighting function in the form given by (25). It computes factor 1 from the input channel frequency and *boftb*. Since we assume a spherically stratified atmosphere, $T(s) = T(z)$ and $\alpha(s) = \alpha(z)$, so the quantity inside the braces comes directly from *bakgrnd, boft, tauprof,* and *boftatm*. *Weight_RTE* computes the factor $\frac{ds}{dz}$ for profile level *i* as

$$\frac{ds}{dz} = \frac{ds(i)}{z_i - z_{i-1}}$$

except for level 1, where *ds* and *dz* are both zero by definition. At that level, *Weight_RTE* uses the approximation

$$\frac{ds}{dz} = csc\ (angle)$$

The last term inside the brackets of factor 2, which is zero for all parameters except temperature, is computed from (26).

Weight_RTE approximates $\frac{\partial\alpha}{\partial x}$ with a two-sided numerical partial derivative. The specified parameter is perturbed by a small fraction (*dx*) of the parameter value (x_i) at each profile level *i*. *Weight_RTE* uses .001 for the temperature and pressure fractions and .01 for the other three parameter fractions. After computing the appropriate value for *dx*, *Weight_RTE* computes the derivative for level *i* as

$$\left(\frac{\partial\alpha}{\partial x}\right)_i = \frac{\alpha^+(s_i) - \alpha^-(s_i)}{dx_i} ,$$

where $\alpha^+(s_i) = \alpha(s_i)$ computed with parameter $x_i = x_i + (0.5 * dx_i)$
$\alpha^-(s_i) = \alpha(s_i)$ computed with parameter $x_i = x_i - (0.5 * dx_i)$

The value for $W_x(z)$ at each profile level is returned in array *wx*.

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